

Solution to Leaving Certificate Maths Problem

$$C = \frac{5t}{1 + \left(\frac{t}{k}\right)^2} \quad \text{K20 Max @ } t=6$$

Find 6

$$C = \frac{5t}{1 + \left(\frac{t}{k}\right)^2} = \frac{5t}{\frac{k^2 + t^2}{k^2}} = \frac{5t}{1} \cdot \frac{k^2}{k^2 + t^2} = \frac{5tk^2}{k^2 + t^2}$$

Find: $\frac{dc}{dt}$

$$u = 5tk^2 \quad u = k^2 + t^2$$

$$\frac{du}{dt} = 5tk^2 \quad \frac{du}{dt} = 2t$$

$$\frac{v \left(\frac{du}{dx}\right) - u \left(\frac{dv}{dx}\right)}{v^2} = \frac{(k^2 + t^2)(5k^2) - (5tk^2)(2t)}{(k^2 + t^2)^2}$$

$$\frac{dc}{dt} = \frac{5k^4 + (5t^2k^2) - 10t^2k^2}{(k^2 + t^2)^2} = \frac{5k^4 - 5t^2k^2}{(k^2 + t^2)^2} = \frac{5k^2(k^2 - t^2)}{(k^2 + t^2)^2}$$

$$\text{Let } \frac{dc}{dt} = 0 \quad \frac{5k^2(k^2 - t^2)}{(k^2 + t^2)^2} = 0$$

$$5k^2(k^2 - t^2) = 0$$

$$5k^2 = 0$$

$$k^2 = 0$$

$$k^2 \neq 0 \text{ as}$$

$$k > 0$$

$$K^2 - t^2 = 0$$

$$K^2 = t^2$$

$$K^2 = 36 \text{ @}t=6$$

$$K = \pm 6$$

$$K \neq 6 \text{ as } k > 0$$

$$K = 6$$